

## Parity violating cylindrical shell in the framework of QED

This article has been downloaded from IOPscience. Please scroll down to see the full text article.

2008 J. Phys. A: Math. Theor. 41 075403

(<http://iopscience.iop.org/1751-8121/41/7/075403>)

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

Download details:

IP Address: 171.66.16.153

The article was downloaded on 03/06/2010 at 07:28

Please note that [terms and conditions apply](#).

# Parity violating cylindrical shell in the framework of QED

I V Fialkovsky<sup>1</sup>, V N Markov<sup>2</sup> and Yu M Pismak<sup>1</sup>

<sup>1</sup> Department of Theoretical Physics, State University of Saint-Petersburg, St Petersburg, Russia

<sup>2</sup> Department of Theoretical Physics, Petersburg Nuclear Physics Institute, St Petersburg, Russia

E-mail: [ignat.fialk@paloma.spbu.ru](mailto:ignat.fialk@paloma.spbu.ru), [markov@thd.pnpi.spb.ru](mailto:markov@thd.pnpi.spb.ru) and [pismak@jp7821.spb.edu](mailto:pismak@jp7821.spb.edu)

Received 22 October 2007, in final form 27 December 2007

Published 5 February 2008

Online at [stacks.iop.org/JPhysA/41/075403](http://stacks.iop.org/JPhysA/41/075403)

## Abstract

We present calculations of Casimir energy (CE) in a system of quantized electromagnetic (EM) field interacting with an infinite circular cylindrical shell (which we call ‘the defect’). Interaction is described in the only QFT-consistent way by Chern–Simon action concentrated on the defect, with a single coupling constant  $a$ . For the regularization of UV divergencies of the theory, we use the Pauli–Villars regularization of the free-EM action. The divergencies are extracted as a polynomial in the regularization mass  $M$ , and they renormalize the classical part of the surface action. We reveal the dependence of CE on the coupling constant  $a$ . Corresponding Casimir force is attractive for all values of  $a$ . For  $a \rightarrow \infty$ , we reproduce the known results for CE for perfectly conducting cylindrical shell first obtained by DeRaad and Milton. As a future task for solving existing arguments on observational status of (rigid) self-pressure of a single object, we propose for investigation a system which we call ‘Casimir drum’.

PACS number: 12.20.Ds

## 1. Introduction

Predicted in 1948 [1] the Casimir effect has been for a long time a non-detectable theoretical ‘play of mind’. Prophesying an attractive force between two neutral parallel conducting planes in vacuum, the Casimir effect is a pure quantum one—there is no such force between the planes in classical electrodynamics.

Development of the experimental technique allowed first to observe the Casimir effect [2] and then to measure it with 5–10% accuracy [3]. Nowadays, due to works of Mohideen and his colleagues, as well as many other groups, the total experimental error for the force between metal surfaces is reduced to 0.5% [4–6].

Original Casimir's configuration of parallel (perfectly conducting) planes is well studied both theoretically and experimentally. Still there are considerable lacunas in our understanding of the effect for complicated geometries and non-perfect materials. The particular interest for cylindrical geometry which we consider in this paper is motivated by the rapid development of the carbon nanotubes technology. For the discussion of possible role of the Casimir force in the dynamics and stability of micro- and nano-electromechanical devices (MEMS and NEMS), see [8, 9] and reference therein.

From the theoretical point of view, the Casimir force between distinct bodies is well established, especially taking into account the main achievements (both analytical and numerical) of the recent time [11–14]. On the other hand, calculations of the Casimir effect for a single object (self-energy, self-stress, etc) still provoke controversies [15, 16], and a self-consistent description of systems with sharp material boundaries is yet to be developed in the framework of the quantum field theory (QFT). In this paper, we address both of these issues.

The paper is organized as follows. In section 2, we discuss the construction of the model for the system of quantum electrodynamics' (QED) fields interacting with the cylindrical shell. In section 3, we sketch our approach to the calculation of the modified propagator of the system and the Casimir energy, and give their explicit form. In section 4, we present the conclusions and discuss the perspectives of the work.

## 2. Statement of the problem

There are several ways to model the presence of matter (which we also call 'spatial defect') in QFT. The simplest one is to fix the values of quantum fields and/or their derivatives with boundary conditions (BC) on the surface of the defect. However, imposing BC is physically unjustified as it constraints all modes of the fields. At the same time, in reality field's modes with high enough frequencies propagate freely through any material boundary.

The most natural generalization of BC is to couple the quantum fields to the classical external field (background) supported spatially on the defect. The simplest case of such background is a singular one with delta-function profile. Its introduction into the classical action is equivalent to imposing matching conditions on the quantum fields which model semitransparent boundaries. Delta-potential is an effective way to describe a thin film present in the system, when its thickness is negligible compared to the distances in the range of interest. In a certain limit (usually the strong coupling one), delta-potentials reproduce simplest BC such as Dirichlet, Neumann or Robin ones in the case of scalar fields [19].

In the context of QFT, such an interaction of quantum fields with delta-potentials (introduced as a part of the action) must be constructed satisfying the basic principles of the theory—locality, gauge and Lorenz invariance, renormalizability. For the first time, this issue was addressed and thoroughly studied (for the case of massless scalar fields) by Symanzik [19]. Since then, there were a number of Casimir calculations with delta-potentials, see, for instance [20–23]. However, until very lately the issue of renormalizability of a theory with delta-potential still invoked contradictions [15, 16], and was apparently resolved in [17]. Still all of the existing papers deal only with simplified scalar models, usually in lower dimensions. In a limited number of particular cases scalar fields can be combined as the TM and TE modes [24] of EM field, to describe some specific aspects of Casimir problems in QED. Until [25, 26] there were no attempts to construct a QED model with a delta-potential interaction satisfying all QFT principles and allowing one to describe self-consistently all possible observable consequences of the presence of a defect. In this paper, we generalize the results of [25] to the case of cylindrical geometry.

Following the approach of [25], we construct a QED model with photon field coupled to the defect through a delta-potential supported on the surface of an infinite circular cylinder. We neglect the interaction of fermion fields with the defect since any observable consequences of such interaction are exponentially suppressed at the distances larger than the inverse electron mass  $m_e^{-1} \approx 10^{-10}$  cm [26, 27]. Thus, massive fermion fields cannot contribute to the Casimir force which has macroscopical (experimentally verified) values at the scale of 10–100 nm. Therefore, we can neglect Dirac fields and consider pure photodynamics.

For the photon field  $A_\mu$  and defect surface described by equation  $\Phi(x) = 0$ , we construct the action as a sum

$$S = S_0 + S_{\text{def}} \quad (1)$$

of the usual Maxwell action of electromagnetic field (throughout the paper, we set  $c = \hbar = 1$ )

$$S_0 = -\frac{1}{4} \int d^4x F_{\mu\nu}(x) F^{\mu\nu}(x), \quad F_{\mu\nu}(x) = \partial_\mu A_\nu - \partial_\nu A_\mu \quad (2)$$

and defect action of Chern–Simon type [28]

$$S_{\text{def}} = \frac{a}{2} \int d^4x \varepsilon^{\mu\nu\rho\sigma} \partial_\mu \Phi(x) \delta(\Phi(x)) A_\nu(x) F_{\rho\sigma}(x). \quad (3)$$

Here,  $\varepsilon_{\mu\nu\rho\sigma}$  is the totally antisymmetric tensor ( $\varepsilon_{0123} = 1$ ) and  $a$  is a dimensionless coupling constant. For a cylinder of radius  $R$  placed along the  $x_3$  axis, we have

$$\Phi(x) = x_1^2 + x_2^2 - R^2. \quad (4)$$

We must stress here that the form of the defect action (3) is completely determined by the above-mentioned basic principles of QFT. In particular, the introduction of any other local, gauge and Lorenz invariant terms (with higher derivatives, etc) unavoidably brings to the theory coupling constants of negative dimensions. Such theories have infinite number of primitively divergent diagrams, being unrenormalizable in conventional sense [19, 29]. Thus, we are left with the Chern–Simon defect action that is space parity violating and this unusual property quite naturally arises at the very beginning of our consideration. We will show below that in the limit of  $a \rightarrow \infty$  the Casimir energy for perfectly conducting cylindrical shell is reproduced.

### 3. Casimir energy and photon propagator

All properties of a QFT system can be described if its generating functional is known

$$G(J) = \mathcal{N} \int \mathcal{D}A \exp\{iS + JA\}. \quad (5)$$

To render the theory UV-finite, instead of  $S$  (1) we set into  $G(J)$

$$S = S_0^{\text{reg}} + S_{\text{def}}, \quad (6)$$

where we introduced Pauli–Villars [30] regularization of the theory

$$S_0^{\text{reg}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - M^{-2} \partial_\lambda F_{\mu\nu} \partial^\lambda F^{\mu\nu}; \quad (7)$$

here and below we omit the sign of integration,  $S_{\text{def}}$  is defined in (3). In the limit  $M \rightarrow \infty$  we return to action  $S$  (1), but then the theory possesses both standard UV divergencies of QED and specific (geometry-dependent) ones in the vacuum loops—in the Casimir energy in particular. These divergences can be canceled by counter terms in the framework of renormalization procedure. If  $M$  is finite there are no divergences in the model with the action  $S$ .

For the definition of the normalization constant  $\mathcal{N}$  in (5), we use the following condition:

$$G(0)|_{a=0} = 1 \implies \mathcal{N}^{-1} = \int \mathcal{D}A \exp\{i\mathcal{S}|_{a=0}\}, \quad (8)$$

which means that in pure photodynamics without a defect  $\ln G(0)$  vanishes. This sets the reference point for the values of the energy density (per unit length of the cylinder) of the system as the latter one is expressed through the value of  $G(0)$

$$\mathcal{E} = -\frac{1}{iT L} \ln G(0), \quad T = \int dx_0, \quad L = \int dx_3. \quad (9)$$

For explicit calculations of  $G(J)$ , we first chose the coordinate basis associated with the cylindrical geometry and transform the vectors accordingly

$$A = (A_0, A_1, A_2, A_3) \rightarrow \bar{A} \equiv \tau A = (A_0, A_\perp, A_\parallel, A_3),$$

with

$$\tau = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \phi & \sin \phi & 0 \\ 0 & -\sin \phi & \cos \phi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad (10)$$

where  $\phi$  is the polar angle in the  $x_1x_2$  plane. After such a linear transformation of integration variables in (5), the defect action (3) does not contain anymore neither the vector components  $A_\perp$  nor the derivatives  $\partial_\perp$  transversal to the defect. This makes it possible to represent the defect action contribution as an integral over auxiliary fields defined on the defect surface only. Then, functional integration becomes purely Gaussian and for  $G(J)$  one calculates explicitly

$$G(J) = (\text{Det } Q)^{-1/2} \exp\{\frac{1}{2}J(D + \tau^T \bar{\mathcal{H}} \tau)J\}. \quad (11)$$

Here,  $\mathcal{D} \equiv \mathcal{D}_{\mu\nu}(x)$  is the standard (UV-regularized) free-photon propagator,  $\bar{\mathcal{H}}$  defines its corrections due to the presence of the defect and  $Q$  describes the dependence of the Casimir energy on the geometry of the defect. In Fourier components respecting the symmetries of the system, the propagator in Feynman gauge can be written as

$$\begin{aligned} \mathcal{D}_{\mu\nu}(x, y) &= \delta_{\mu\nu} \int \frac{dp_S}{(2\pi)^3} e^{ip_S(x_S - y_S)} D(p_S; \rho_x, \rho_y) \\ D(p_S; \rho_x, \rho_y) &= I_n(\chi \rho_{<}) K_n(\chi \rho_{>}) - I_n(q_S \rho_{<}) K_n(q_S \rho_{>}), \\ \chi &= i\sqrt{p_0^2 - p_3^2}, \quad q_S = \sqrt{M^2 + \chi^2}, \end{aligned} \quad (12)$$

where  $x_S = (x_0, \phi R, x_3)$ ,  $p_S = (p_0 n/R, p_3)$ ,  $\int dp_S = \int dp_0 dp_3 \sum_{n=-\infty}^{\infty}$ ,  $I_n, K_n$  are the modified Bessel functions,  $\rho_{<,>}$  is smaller (bigger) of  $\rho_{x,y}$  and  $\rho_x = \sqrt{x_1^2 + x_2^2}$  is the polar radius in the plane  $x_1x_2$ . In the same Fourier representation for  $\bar{\mathcal{H}}(x, y)$  and  $Q(x_S, y_S)$ , we can write

$$\bar{\mathcal{H}}_{ab}(p_S; \rho_x, \rho_y) = aD(p_S; \rho_x, R) L_{ac} Q_{cb}^{-1}(p_S) D(p_S; R, \rho_y), \quad (13)$$

$$Q_{ab}(p_S) = \delta_{ab} + aD(p_S; R, R) L_{ab}, \quad (14)$$

$$L_{ab} = 2iR \varepsilon_{abc} p_S^c, \quad a, b, c = 1, 2, 3. \quad (15)$$

Using the famous  $\text{tr} \ln = \ln \det$  identity, one can express the Casimir energy (9) as

$$\mathcal{E} = \frac{1}{2iT L} \int dx_S \text{tr}(\text{Ln } Q(x_S - y_S))_{x_S=y_S} = \frac{1}{8\pi^2} \int dp_S \text{tr} \ln Q(p_S), \quad (16)$$

where  $\text{tr} \ln Q(p_S)$  denotes the sum of diagonal elements of the  $3 \times 3$  matrix  $\ln Q(p_S)$ . Putting (14) into (16), one writes for the energy density

$$\begin{aligned} \mathcal{E} &= \frac{1}{4\pi R^2} \int_0^\infty p \, dp \sum_{n=-\infty}^\infty \ln(1 + a^2 Y_n^M(p)) \\ Y_n^M(p) &= -4(I_n(p)K_n(p) - I_n(q)K_n(q)) \\ &\quad \times \left( p^2(I_n'(p)K_n'(p) - I_n'(q)K_n'(q)) + \frac{n^2(RM)^2}{p^2 + (RM)^2} I_n(q)K_n(q) \right), \end{aligned} \tag{17}$$

where  $q = \sqrt{p^2 + (RM)^2}$ .

It is easy to check that (17) is finite for any fixed value of auxiliary mass  $M$  and diverges in the limit  $M \rightarrow \infty$ . With the help of uniform (Debye) asymptotics of the Bessel functions [31], we subtract the most divergent terms from the integrand to make it finite when regularization is removed, and add them explicitly. Following then the renormalization procedure, we extract from the subtraction exact (polynomial and/or logarithmic) dependence on  $M$ , and construct the counter terms. This is done with the help of the generalized Abel–Plana formula [32] and its modification [33].

As a result of the calculations, we present the energy density (17) as a sum

$$\mathcal{E} = \mathcal{E}_{\text{Cas}} + \Delta \tag{18}$$

of finite Casimir energy

$$\begin{aligned} \mathcal{E}_{\text{Cas}} &= \frac{1}{4\pi R^2} \left( \frac{a^2 \ln 2\pi}{4(1+a^2)} + \int_0^\infty p \, dp \epsilon^{\text{finite}} \right), \\ \epsilon^{\text{finite}} &= \ln \left( \frac{1+a^2 Y_0(p)}{1+a^2} \right) + \frac{a^2}{1+a^2} \frac{p^4}{4(1+p^2)^3} \\ &\quad + 2 \sum_{n=1}^\infty \left( \ln \left( \frac{1+a^2 Y_n(p)}{1+a^2} \right) + \frac{a^2}{1+a^2} \frac{p^4}{4(n^2+p^2)^3} \right); \end{aligned} \tag{19}$$

here,  $Y_n(p) = \lim_{M \rightarrow \infty} Y_n^M(p) = -4p^2 I_n(p)K_n(p)I_n'(p)K_n'(p)$ , and the counter terms

$$\Delta = RM^3 A_3 + \frac{M}{R} A_1, \tag{20}$$

where

$$A_1 = \frac{2a^2}{\pi(1+a^2)} \int_0^\infty p \, dp \int_0^\infty dn y_n^{(1)}(p), \quad A_3 = \frac{1}{2\pi} \int_0^\infty p \, dp \int_0^\infty dn \ln(1 + a^2 y_n^{(0)}(p)),$$

with

$$y_n^{(0)} = 2 - (n^2 + p^2 + 1)^{-1} - 2\sqrt{n^2 + p^2} / \sqrt{n^2 + p^2 + 1}$$

and  $y_n^{(1)}(p)$  is the first-order term of uniform asymptotic of  $Y_n(p)|_{M=1}$ .

$\mathcal{E}_{\text{Cas}}$  as the function of  $a$  is real for  $a \in \Re$  and for  $a \in i(-1, 1)$ . The latter region is out of physical interest [21] as for this case the actions (1) and (6) acquire the imaginary part. For all physically sensible values of  $a$ ,  $\mathcal{E}_{\text{Cas}}$  is positive giving rise to attractive Casimir force. In the limit  $a \rightarrow \infty$ , one can easily derive that (19) coincides explicitly with known results for the Casimir energy of a perfectly conducting cylinder [7].

For the renormalization of the counter terms (20), we must introduce into action (6) also the classical energy density

$$E = R\sigma_0 + \frac{h_0}{R},$$

where the bare parameters  $\sigma_0$  and  $h_0$  are the surface tension and inverse radius parameter, correspondingly. To renormalize the divergencies, we make the following redefinition:

$$\sigma_0 = \sigma - M^3 A_3, \quad h_0 = h - M A_1, \quad (21)$$

where  $\sigma$  and  $h$  must be taken as the ‘input’ parameters to the theory in the spirit of [17]. They describe the properties of material of the defect. Just as the electron mass  $m$  and charge  $e$  in QED, the values of  $\sigma$  and  $h$  cannot be predicted from the theory and must be determined from appropriate experiments. Thus, in addition to the standard QED normalization conditions, one needs three additional independent experiments to remove all the ambiguities of our model—to determine  $\sigma$  and  $h$ , and to set the scale of the coupling constant  $a$ .

#### 4. Conclusions

We constructed the QED model which describes a photon field interacting with semitransparent cylindrical shell (two-dimensional defect surface). The form of interaction—Chern–Simon action—is completely determined by the basic principles of QFT: locality, gauge and Lorenz invariance, renormalizability. The defect action is parity odd and P-transformation is equivalent to the change of sign of the defect coupling constant  $a$ . We calculated explicitly the modified photon propagator and the Casimir energy. The latter one appears to be P-invariant being an even function of  $a$  and tends with  $a \rightarrow \infty$  to its value for perfectly conducting shell. Parity violation manifests itself only when external field is applied [25]. Thus, we can say that such defect action mimics the behavior of thin films of magnetoelectrics.

Consideration of two-dimensional defects within the scope of renormalizable QFT is justified by existing experimental data. It unambiguously shows that the Casimir force for objects with sharp boundaries has  $1/r^3$  ( $r$  is the distance between the objects) behavior, and thus is governed by the dimensionless parameters of the system. On the other hand, from obvious dimensional reasoning it follows that the presence of any dimensional constants, such as finite width of the film  $h$ , finite conductivity  $\delta$  or final UV cut-off in non-renormalizable models, can only give corrections in the order  $h^2/r^5$ ,  $\delta^2/r^5$  or higher. Thus, to the next to leading order in inverse powers of  $r$ , we can stay with surface contributions only.

Effectively, with the defect action, we model the interaction of electromagnetic field with a surface layer (thin film) of atoms constituting the media, which can naturally be parity violating. This property of the media translates into the surface properties described in our model by a single constant  $a$ . The parity in our model is preserved in two cases: the trivial one  $a = 0$ , and  $a \rightarrow \infty$  that corresponds to the perfect conductor limit as we showed above. We predict that thin films which are parity even should have universal amplitude of the Casimir force: either vanishing (with  $a = 0$ ) or coinciding with one of the perfect conductor ( $a = \infty$ ). From the theoretical point of view, we cannot decide whether materials with finite defect coupling  $a$  exist or not.

The presented calculations of the Casimir energy (and subsequently the Casimir force) indirectly presume that the energy change is measured between two adiabatic states of the system which differ by the radius of the cylinder. In particular, this means that if an experiment is to be carried out, the cylinder must be deformed as a whole, uniformly along all of its (infinite) length. Such an experimental setup looks unrealistic but is still possible in principle. This rises arguments in the literature (see review [34]) that calculations of a renormalized (rigid) self-pressure has insignificant (if any) predictive power.

As a way to resolve contradictions, one has to calculate ‘soft’ self-pressure for local deformations of the shape of the body and reveal its dependence on the position along the body’s surface. For example, one can consider a perfectly conducting cylinder shell of finite

length  $L$  and radius  $R$ . It is clear that in the limit  $R/L \rightarrow \infty$  in the vicinity of the cylinder axis the local self-pressure must reproduce the attractive Casimir force between parallel plates—the two bases of the cylinder. On the other hand, when  $R/L \sim 1$  one can consider the deformations of the cylinder as a whole, and should recover the rigid stress. We call such system a Casimir drum. Explicit calculation of the Casimir energy, rigid and soft self-pressures of the Casimir drum is the next step for our research.

## Acknowledgments

Authors would like to thank Professors R Jaffe and D Vassilevich for fruitful discussions on the subject of the paper. V N Markov and Yu M Pismak are grateful to the Russian Foundation of Basic Research for financial support (RFRB grant 07-01-00692). Yu M Pismak was supported in part by the Swiss National Science Foundation (grant Pl012-1189933) and under the Russian Presidential programme Leading Scientific School.

## References

- [1] Casimir H B G 1948 *Proc. K. Ned. Akad. Wet.* **51** 793
- [2] Sparnaay M J 1958 *Physica* **24** 751
- [3] Lamoreaux S K 1997 *Phys. Rev. Lett.* **78** 5
- [4] Mohideen U and Roy A 1998 *Phys. Rev. Lett.* **81** 4549
- [5] Decca R S, Fischbach E, Klimchitskaya G L, Krause D E, Lopez D and Mostepanenko V M 2003 *Phys. Rev. D* **68** 116003
- [6] Klimchitskaya G L, Decca R S, Fischbach E, Krause D E, Lopez D and Mostepanenko V M 2005 *Int. J. Mod. Phys. A* **20** 2205
- [7] DeRaad L L Jr and Milton K 1981 *Ann. Phys., N.Y.* **136** 229  
Milton K A, Nesterenko A V and Nesterenko V V 1999 *Phys. Rev. D* **59** 105009 (Preprint hep-th/9711168v3)
- [8] Barcenas J, Reyes L and Esquivel-Sirvent R 2005 *Appl. Phys. Lett.* **87** 263106
- [9] Lin W-H and Zhao Y-P 2007 *J. Phys. D: Appl. Phys.* **40** 1649
- [10] Gies H and Klingmuller K 2006 *Phys. Rev. Lett.* **96** 220401 (Preprint quant-ph/0601094)
- [11] Emig T, Jaffe R L, Kardar M and Scardicchio A 2006 *Phys. Rev. Lett.* **96** 080403
- [12] Bordag M 2006 *Phys. Rev. D* **73** 125018 (Preprint hep-th/0602295)
- [13] Emig T, Graham N, Jaffe R L and Kardar M 2007 Casimir forces between arbitrary compact objects  
Preprint 0707.1862
- [14] Kenneth O and Klich I 2007 Casimir forces in a T operator approach Preprint 0707.4017
- [15] Graham N, Jaffe R L, Khemani V, Quandt M, Scandurra M and Weigel H 2003 *Phys. Lett. B* **572** 196 (Preprint hep-th/0207205)  
Jaffe R L 2003 *AIP Conf. Proc.* **687** 3 (Preprint hep-th/0307014)  
Graham N, Jaffe R L, Khemani V, Quandt M, Schroeder O and Weigel H 2004 *Nucl. Phys. B* **677** 379 (Preprint hep-th/0309130)
- [16] Milton K A 2004 *J. Phys. A: Math. Gen.* **37** 6391–406 (Preprint hep-th/0401090)  
Milton K A 2004 Invited talk given at Marcel Grossmann X (Preprint hep-th/0401117)
- [17] Bordag M and Vassilevich D V 2004 *Phys. Rev. D* **70** 045003 (Preprint hep-th/0404069v2)
- [18] Milton K A 2004 *J. Phys. A: Math. Gen.* **37** R209 (Preprint hep-th/0406024v1)
- [19] Symanzik K 1981 *Nucl. Phys. B* **190** 1
- [20] Bordag M, Kirsten K and Vassilevich D 1999 *Phys. Rev. D* **59** 085011 (Preprint hep-th/9811015v1)
- [21] Scandurra M 2000 *J. Phys. A: Math. Gen.* **33** 5707–18 (Preprint hep-th/0004051v1)  
Scandurra M 2000 *PhD Thesis* (Preprint hep-th/0011151v2)
- [22] Graham N, Jaffe R L and Weigel H 2002 *Int. J. Mod. Phys. A* **17** 846–69 (Preprint hep-th/0201148v1)  
Graham N, Jaffe R L, Khemani V, Quandt M, Scandurra M and Weigel H 2002 *Nucl. Phys. B* **645** 49–84 (Preprint hep-th/0207120v2)
- [23] Milton K A 2003 *Phys. Rev. D* **68** 065020 (Preprint hep-th/0210081v2)
- [24] Stratton J A 1941 *Electromagnetic Theory* (New York: McGraw-Hill)
- [25] Markov V N and Pis'mak Yu M 2005 Preprint hep-th/0505218  
Markov V N and Pis'mak Yu M 2006 *J. Phys. A: Math. Gen.* **39** 6525–32 (Preprint hep-th/0606058)



- [26] Fialkovsky I V, Markov V N and Pis'mak Yu M 2006 *Int. J. Mod. Phys. A* **21** 2601–16 (Preprint [hep-th/0311236](#))
- [27] Fialkovsky I V, Markov V N and Pis'mak Yu M 2006 *J. Phys. A: Math. Gen.* **39** 6357–63
- [28] Chern S S 1971 *Proc. Natl Acad. Sci. USA* **68** 791  
Carrol S N, Field G B and Jackiw R 1990 *Phys. Rev. D* **41** 1231
- [29] Collins J C 1984 *Renormalization* (Cambridge: Cambridge University Press)
- [30] Pauli W and Villars F 1949 *Rev. Mod. Phys.* **21** 434–44
- [31] Abramowitz M and Stegun I A (ed) 1972 *Handbook of Mathematical Functions* (NBS)
- [32] Saharian A A 2000 IC/2000/14 Preprint [hep-th/0002239](#)  
Saharian A A 2007 Preprint [0708.1187v1](#)
- [33] Fialkovsky I V 2007 Preprint [0710.5539v1](#)
- [34] Milton K A 2004 *J. Phys. A: Math. Gen.* **37** R209 (Preprint [hep-th/0406024v1](#))